# Time series analysis: from econometrics to epidemiology 

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## Time series: definition

A time series is defined as a collection of observations sampled at equally-spaced and ordered time points

Statistically, the series is treated as a sequence of $n$ random variables $Y_{1}, \ldots, Y_{t}, \ldots, Y_{n}$, assumed to be a single realization of a discrete-time stochastic process $\left\{Y_{t}\right\}$

## ARIMA models

Many probabilistic models are based on the assumption of (weakly) stationarity of the series:

- Constant $\mu=\mathrm{E}(Y)$
- $\operatorname{Cov}\left(Y_{t}, Y_{s}\right)=\gamma(h)$, with $h=|t-s|$

Series usually exhibit stochastic or deterministic trends
Stationarity may be recovered by auto-regressing, integrating, and averaging (filtering) the series $\longrightarrow$ ARIMA models

## Temporal decomposition

In epidemiological studies, the purpose of time series analysis is shifted from prediction to estimation

The series is decomposed into long-time and seasonal trends (or other components related to different timescales), and the contribution of additional terms

The series $\left\{Y_{t}\right\}$ is then described as the sum of deterministic signal plus a stochastic stationary noise

## Temperature and mortality series

New York 1987-2000



## Regression models

In modern applications, decomposition is performed through regression models

A general model to describe the series of observed outcomes $y_{t}$, with $t=1, \ldots, n$ is given by:

$$
g\left(y_{t}\right)=\alpha+\sum_{j=1}^{J} s_{j}\left(x_{t j} ; \boldsymbol{\beta}_{j}\right)+\epsilon_{t}
$$

Focus on the index $t$ : temporal structure of the association

## Methodological research topics

- Methods to describe the temporal structure of the association (delayed effects)
- Smoothing techniques for control of seasonality
- Methods to incorporate residual correlation
- Interrupted time series: before-after design


## Delayed effects



Time (Lags)

## Distributed lag (non-linear) models

A statistical framework to describe simultaneously non-linear and delayed effects in time series data

DLNMs are expressed by the definition of a cross-basis:
bi-dimensional function describing the relationship along the spaces of predictor and lag

This framework is implemented in the R package dlnm

## Temperature and mortality (I) <br> Chicago 1987-2000



## Temperature and mortality (II)

## Chicago 1987-2000



Temperature $=-10$





